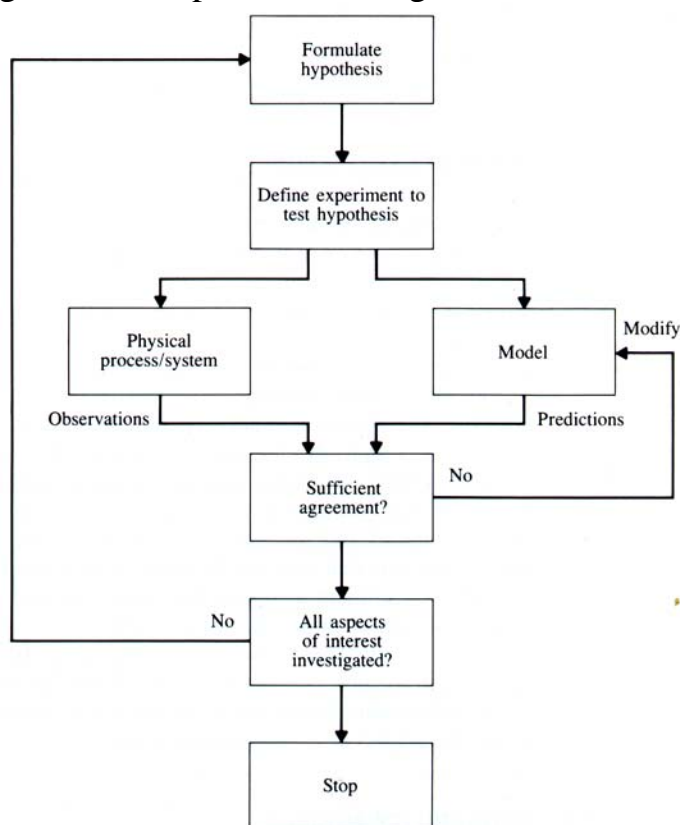


## Chapter 1 – Probability Models

### Mathematical Models

- **Experiments:** Costly way of testing a design or solve a problem.
- **Model:** Approximate representation of a physical situation.
- **Useful Model:** Able to explain all relevant aspects of a given phenomenon.
- **Mathematical Models:** If observational phenomenon has measurable properties then a mathematical model consisting of a set of assumptions about the system is employed.

Conditions under which an experiment is performed and a model is assumed are very critical. Change the assumptions then a “good” model can be a great fiasco.



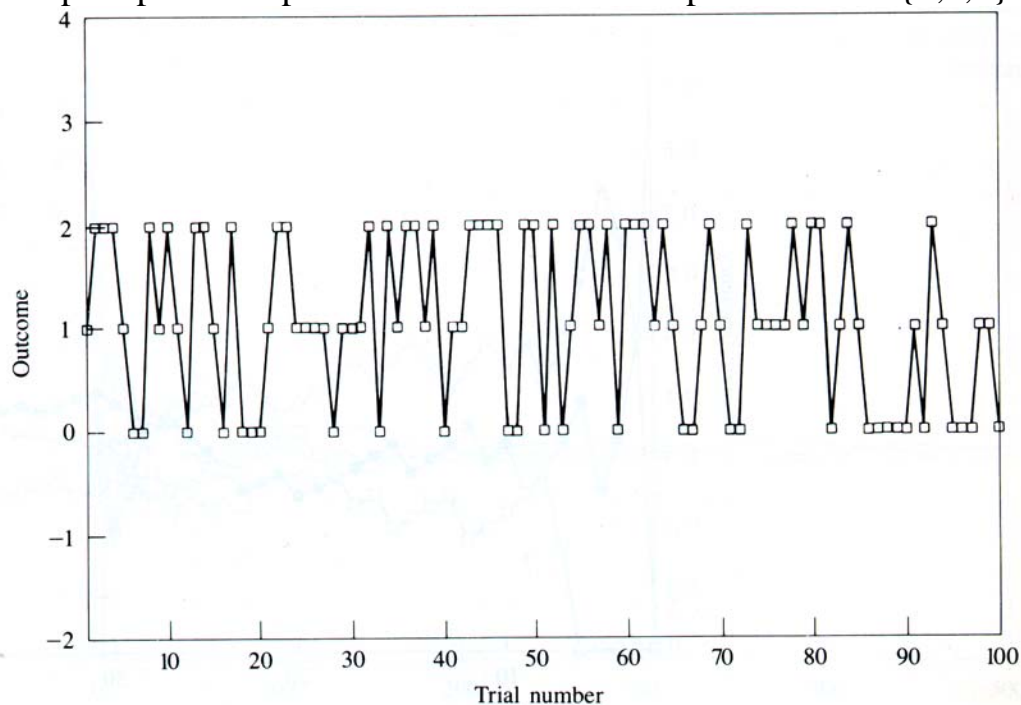
- **Computer Simulation Models:** They mimic or simulate the dynamics of a system
- **Deterministic Models:** Lab and textbook cases, conditions determine outcome
  1. Circuit Theory
  2. Ohm's Law
  3. Kirchoffs' Laws
  4. Transforms: FFT; Laplace Transforms
  5. Convolution :Input/output behavior of systems with well-defined coefficients

- Probabilistic (Stochastic, Random) Models: involve phenomena that exhibit unpredictable variation and randomness.

**Ex: Urn with three balls; (0,1,2 marked)**

-- Outcome: A number from the set {0,1,2}

-- Sample Space: All possible outcomes of an experiment:  $S = \{0,1,2\}$



**Statistical Regularity:**

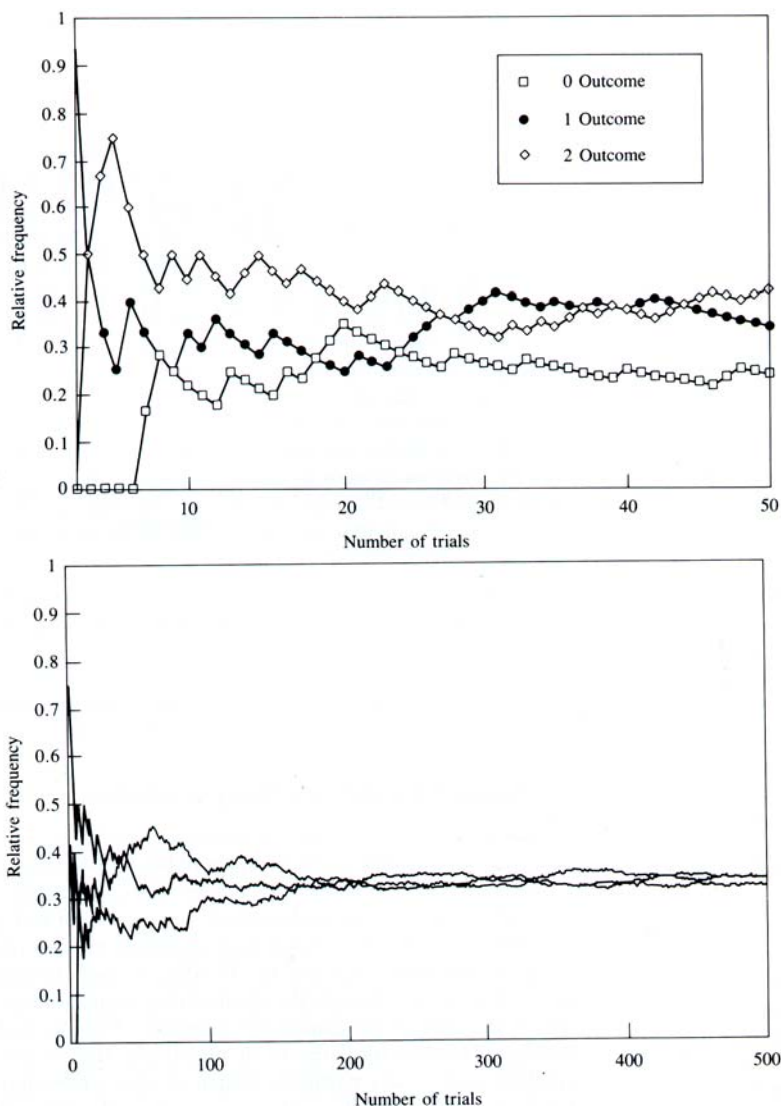
Relative Frequency: 
$$f_k(n) = \frac{N_k(n)}{n}$$

$n$ : # of experiments under identical conditions;

$k$ : Counter index

$$\left. \begin{array}{l} N_0: \# \text{ of "0" balls} \\ N_1: \# \text{ of "1" balls} \\ N_2: \# \text{ of "2" balls} \end{array} \right\} \text{ in } n \text{ tries}$$

Let  $n \rightarrow \infty$  then  $\lim_{n \rightarrow \infty} f_k(n) = P_k$ ; where  $P_k$ : probability of the outcome  $E_k$ .



### Properties of Relative Frequency:

Suppose a random experiment has  $K$  possible outcomes:  $S = \{1, 2, \dots, K\}$ . Then in “ $n$ ” trials we have

$$0 \leq N_k(n) \leq n \quad \text{for } k = 1, 2, \dots, K \Rightarrow 0 \leq f_k(n) \leq 1$$

and

$$\sum_{k=1}^K N_k(n) = n \quad \text{which implies:} \quad \sum_{k=1}^K f_k(n) = 1$$

**Event**  $\equiv$  Any outcome of an experiment satisfying certain condition(s).

**Ex:** Consider the 3-ball urn experiment

A: even = {0,2} then,

$$f_A(n) = \frac{N_0(n) + N_2(n)}{n} = f_0(n) + f_2(n)$$

**Disjoint (mutually exclusive) events:** If A or B can occur but not both, then

$$f_C(n) = f_A(n) + f_B(n)$$

Relative frequency of two disjoint events is the sum of their individual relative frequency.

### Kolmogorov's axioms to form a Theory of Probability: Assumptions:

1. Random experiment has been defined and the sample space S has been identified.
2. A class of subsets of S has been specified.
3. Each event A has been assigned a number P(A) such that,
  1.  $0 \leq P(A) \leq 1$
  2.  $P(S) = 1$
  3. If A and B are mutually exclusive events then
 
$$P(A \text{ or } B) = P(A) + P(B)$$

**Kolmogorov's axioms are sufficient to build a consistent Theory of Probability.**

### Example: Packet Voice Communication system Efficiency

Due to silences voice communication is very inefficient on dedicated lines. It is observed that only "1/3" of the time actual speech goes through. How to increase this rate by using prob. approaches???

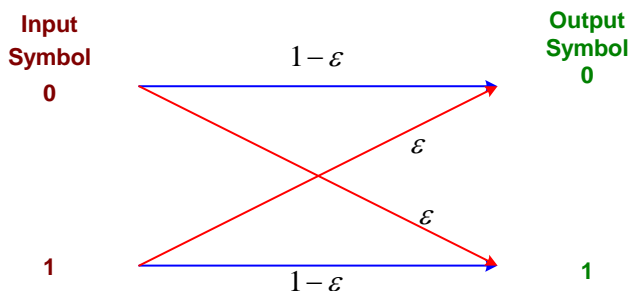
**Solution:** Error vs rate trade off in digital information (BCS) transmission/storage

$$\left. \begin{array}{l} 0 \rightarrow 000 \\ 1 \rightarrow 111 \end{array} \right\} + \text{Majority Rule to make decision at the receiver.}$$

If each bit has a  $P_e = 10^{-3}$  due to this simple scheme,  $P_e \rightarrow 3 \times 10^{-6}$ .

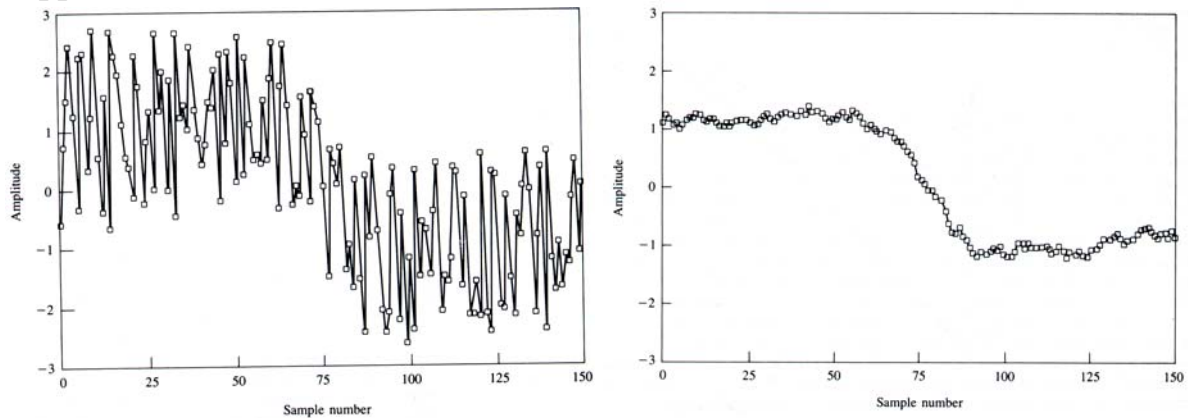
**Cost:** Rate is decrease to 1/3 of the original.

Binary Symmetric Channel with cross-over probability:  $\varepsilon = P_e = 10^{-3}$

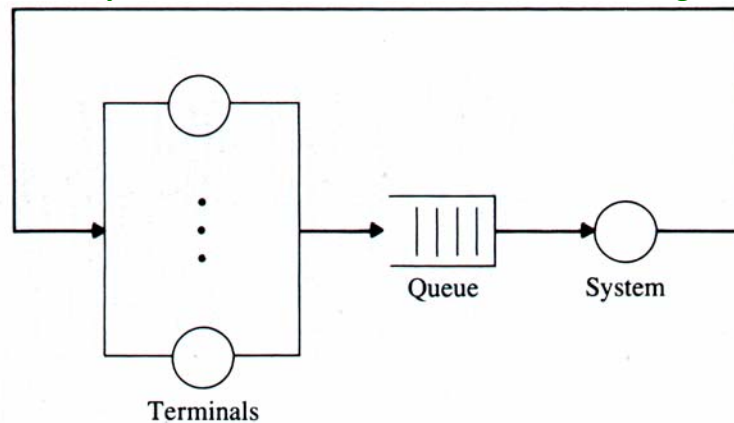


### Example: Signal Enhancement Using Filters

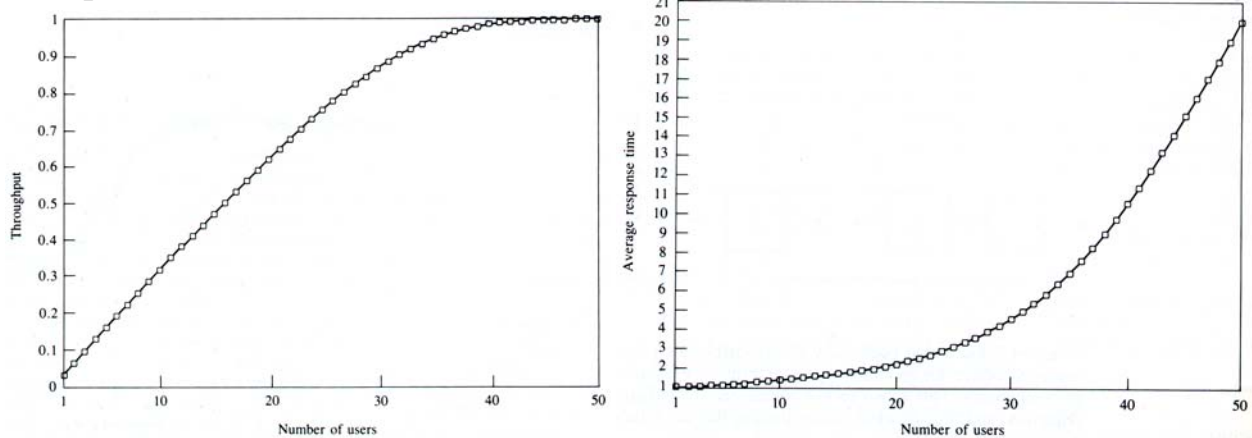
Given a signal  $x(t)$  corrupt with noise and has a Signal-to-Noise Ratio value SNR. If you filter this noisy signal with a properly designed –hopefully adaptive, filter to suppress noise, we obtain an enhanced signal, (smoothed by the filter.)



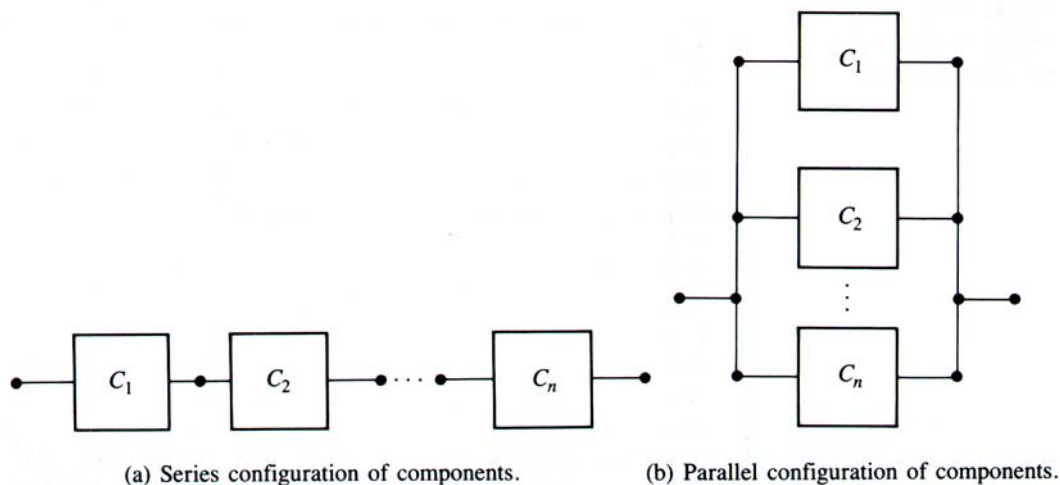
### Example: Multi User Systems with Queues: Resource sharing



Two performance curves for multi-user computer queuing studies:



### Example: System Reliability: Cascade vs. Parallel Systems



**Issues:** Need of a clock vs. the system delay or throughput rate.

#### Ex: Prob. 1.1

Experiment: Selecting two balls in succession from an urn with 2 black + 1 white ball without replacement.

a) Sample Space:  $S = \{bb, bw, wb\}$  If: 1<sup>st</sup> ball is B, then 2<sup>nd</sup> is B or W  
If: 1<sup>st</sup> ball is W, then 2<sup>nd</sup> is only B

b) With replacement: All outcomes are W or B, 2then  
 $S = \{bb, bw, wb, ww\}$

c)  $f_{ww}(n)$  in part (a)?

Not possible to have W then W  $\Rightarrow f_{ww}(n) = 0$

$f_{ww}(n)$  in part (b)?

$N_w(n) = 1/3$ , since 1 W and 2 B balls always

Of these 1/3 outcomes again 1/3 are W since balls are replaced.

$$\therefore f_{ww}(n) = (1/3) * (1/3) = 1/9$$

2<sup>nd</sup> draw is effected by the outcome of 1<sup>st</sup> draw?

a) Yes

No.